Econ4925 Seminar exercise 1

1. Consider the social planning problem

$$\max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\,dz$$

subject to

$$R_t \le R_{t-1} + w_t - e_t^H$$
$$R_t \le \bar{R}$$
$$R_t, e_t^H \ge 0$$
$$T, w_t, R_o, \bar{R} \text{ given, } R_T \text{ free, } t = 1,..,T$$

where

$$e_t^H = \text{electricity production during period } t \text{ (kWh)}$$
$$p_t(e_t^H) = \text{demand function}$$
$$R_t = \text{reservoir level at end of period } t \text{ (kWh)}$$
$$w_t = \text{inflow during period } t \text{ (kWh)}$$
$$\bar{R} = \text{reservoir constraint (kWh)}$$

1. Derive the Kuhn – Tucker conditions for the problem. (Hint: set up the Lagrangian function and differentiate wrt the endogenous variables.) Discuss feasible values within an optimal solution for social prices and water values for period *T*. Discuss the role of an assumption of no satiation of demand in the last period, and in general any period.
2. Dynamic programming problems can be solved by backward induction, starting with solving the problem for the last period as done in problem 1. above. Assume that for all periods in the interval from t+2 to T-1 (including both periods) the reservoir is inbetween full and empty. What will be the social price for these periods going backwards from *T*? (Hint: continue with looking at the 1.order conditions for period *T*-1, etc.)
3. Assume that in period t+1 the reservoir is emptied, $R_{t+1} = 0$. Explain why the social price for period t+1 is greater than the social price for period t+2 if this should be a part of an optimal solution. (Hint: Look at the 1.order condition for the reservoir in period t+1 and the role of the condition $R_{t+1} = 0$.)
4. Assume that for periods from t back to s+1 (with $s+1 < t$) the reservoir has been between full and zero. Furthermore, assume that for period s the reservoir constraint is

binding. Use the Kuhn – Tucker conditions to show that in the general case the water value for period *s* is less than the water value for period *s*+1. (Hint: use the complementary slackness condition for the shadow price on the reservoir capacity constraint.) Assume that the reservoir from period *s*-1 back to period 1 stays between full and zero. Explain why prices and / or water values after period *s* has no influence on the price from *s* back to period 1.

5. Introduce discounting of the objective function by using the discount factor $\beta_t = (1+r)^{-(t-1)}$ where *r* is the rate of discount. Discuss the qualitative impact on the optimal solution for prices looking at the change in price from a period *t* to period *t*+1, when we assume that there is no binding reservoir constraints for neither period t and period *t*+1. Discuss the similarity with Hotelling's rule for depletion of a non-renewable resource.